ICM-9105-09 Field of Values and the ADI Method for Non-Normal Matrices, Gerhard Starke, Linear Algebra and its Applications, (to appear).

ABSTRACT: The convergence of an iterative method is usually measured either by the spectral radius or by a matrix norm, e.g., the spectral norm, of the corresponding iteration operator. However, for non-normal matrices, each one of these approaches has its drawbacks. The first one only describes the asymptotic rate of convergence, i.e., the behavior of the method after a large number of iterations, the second one usually produces estimates which are much too pessimistic. Recently M. Eiermann suggested to use the field of values to judge the performance of an iterative method. His results will be generalized in this paper by upper bounds for the error reduction in the ADI method based on the field of values. In particular, for the stationary ADI method, i.e., using the same parameters in each step, we obtain the error estimate

$$||\mathbf{x}_m - \mathbf{x}||_2 \leq c \frac{\max_{\lambda \in W(V)} |r_m(\lambda)|}{\min_{\mu \in W(-H)} |r_m(\mu)|} ||\mathbf{x}_0 - \mathbf{x}||_2$$

for the commutative case, i.e., if $HV = VH$ is fulfilled.

Any irreducible tridiagonal matrix can be transformed into a complex symmetric tridiagonal matrix by simply replacing the sub- and superdiagonal elements by their geometric mean. We will show that the field of values of a tridiagonal matrix becomes minimal after this transformation. This result can be generalized to Kronecker sums of tridiagonal matrices which occur in many practical applications, e.g., discretized elliptic boundary value problems. Our numerical experiments show that the estimates for the error reduction in the ADI method — applied to the corresponding symmetrized linear system — are useful for an a priori prediction of the convergence behavior. Moreover, the results suggest the use of the field of values as a basis for the calculation of the optimal ADI parameters if H and V are highly non-normal.