

ICM-9106-10 The Laguerre Inequalities with Applications to a Problem Associated with the Riemann Hypothesis, G. Csordas, A. Ruttan and R.S. Varga, Numerical Algorithms, 1 (1991) 305-330.

ABSTRACT: We investigate here a new numerical method, based on the Laguerre inequalities, for determining lower bounds for the de Bruijn-Newman constant Λ , which is related to the Riemann Hypothesis. (Specifically, the truth of the Riemann Hypothesis would imply that $\Lambda \leq 0$.) Unlike previous methods which involved either finding nonreal zeros of associated Jensen polynomials or finding nonreal zeros of a certain real entire function, this new method depends only on evaluating, in real arithmetic, the Laguerre difference

$$L_1(H_\lambda(x)) := (H'_\lambda(x))^2 - H_\lambda(x) \cdot H''_\lambda(x) \quad (x, \lambda \in \mathbb{R}),$$

where $H_\lambda(z) := \int_0^\infty e^{\lambda t^2} \Phi(t) \cos(tz) dt$ is a real entire function. We apply this method to obtain the new lower bound for Λ ,

$$-0.0991 < \Lambda,$$

which improves all previously published lower bounds for Λ .