ICM-9108-11 A Hybrid Arnoldi-Faber Iterative Method for Nonsymmetric Systems of Linear Equations, Gerhard Starke and Richard S. Varga, Numerische Mathematik, (to appear).

ABSTRACT: We present here a new hybrid method for the iterative solution of large sparse nonsymmetric systems of linear equations, say of the form $A\mathbf{x} = \mathbf{b}$, where $A \in \mathbb{R}^{N,N}$, with A nonsingular, and $\mathbf{b} \in \mathbb{R}^N$ are given. This hybrid method begins with a limited number of steps of the Arnoldi method to obtain some information on the location of the spectrum of A, and then switches to a Richardson iterative method based on Faber polynomials. Unlike other hybrid algorithms based on eigenvalue estimates, our numerical experiments indicate that our new hybrid method performs well for non-normal matrices. This is justified theoretically by the fact that Faber polynomials are not only nearly optimal on the domain of estimated eigenvalues of A, but also in neighborhoods of this domain, determined from level sets of some associated conformal mapping. For a polygonal domain, the Faber polynomials can be constructed recursively from the parameters in the Schwarz-Christoffel mapping function. Numerical experiments show that, for non-normal problems, this hybrid algorithm often converges at least as fast as other hybrid algorithms and much faster than restarted versions of the GMRES algorithm. It is, however, sensitive (as other hybrid methods also are) to the amount of information on the spectrum of A acquired during the first (Arnoldi) phase of this procedure.