ICM-9112-22 Construction of Polynomials that are Orthogonal with Respect to a Discrete Bilinear Form, Lothar Reichel, Adv. Comput. Math., 1(1993), pp. 241-258.

ABSTRACT: We describe a new algorithm for the computation of recursion coefficients of monic polynomials $\{p_j\}_{j=0}^{n}$ that are orthogonal with respect to a discrete bilinear form $(f,g) := \sum_{k=1}^{m} f(x_k)g(x_k)w_k$, $m \geq n$, with real distinct nodes x_k and real nonvanishing weights w_k. The algorithm proceeds by applying a judiciously chosen sequence of real or complex Givens rotations to the diagonal matrix diag $[x_1, x_2, \ldots, x_m]$ in order to determine an orthogonally similar complex symmetric tridiagonal matrix T, from whose entries the recursion coefficients of the monic orthogonal polynomials easily can be computed. Fourier coefficients of given functions can conveniently be computed simultaneously with the recursion coefficients. Our scheme generalizes methods by Elhay, Golub and Kautsky [6] based on Givens rotations for updating and downdating polynomials that are orthogonal with respect to a discrete inner product. Our scheme also extends an algorithm for the solution of an inverse eigenvalue problem for real symmetric tridiagonal matrices proposed by Rutishauser [20], Gragg and Harrod [17], and a method for generating orthogonal polynomials based thereon [18]. Computed examples that compare our algorithm with the Stieltjes procedure show the former to generally yield higher accuracy except when $n \ll m$. If n is sufficeintly much smaller than m, then both the Stieltjes procedure and our algorithm yield accurate results.