ICM-9207-39 Acceleration of Relaxation Methods for Non-Hermitian Linear Systems, M. Eiermann, W. Niethammer and R.S. Varga.

ABSTRACT: Let $A = I - B \in \mathbb{C}^{n,n}$, with $\mathrm{diag}(B) = \mathbf{0}$, denote a nonsingular non-Hermitian matrix. To iteratively solve the linear system $A\mathbf{x} = \mathbf{b}$, two splittings of A, together with induced relaxation methods, have been recently investigated in [W. Niethammer and R.S. Varga, *Results in Math.*, 16(1989), pp. 308-320]. The *Hermitian splitting* of A is defined by $A = M^h - N^h$, where $M^h := (A + A^*)/2$ is the Hermitian part of A. The *skew-Hermitian splitting* of A is similarly defined by $A = M^s - N^s$ with $M^s := I + (A - A^*)/2$.

Here, we consider k-step iterative methods to accelerate the relaxation schemes (involving a relaxation factor ω) which are generated by these two splittings. We are not primarily interested in determining the optimal relaxation factor ω which minimizes the spectral radius of the associated iteration operator. Rather, we seek a value of ω such that the resulting relaxation method can be most efficiently accelerated by a k-step method. For the Hermitian splitting, the choice $\omega = 1$ (together with a suitable Chebyshev acceleration) turns out to be optimal in this sense. For the skew-Hermitian splitting, we propose a *hybrid* scheme which is nearly optimal.

As another application of this latter hybrid procedure, we analyse the block-Jacobi method arising from a model equation for a convection-diffusion problem.