

ABSTRACT: The ADI iteration method for the solution of Sylvester's equation  $AX - XB = C$  proceeds by strictly alternating between the solution of the two equations

$$\begin{aligned}(A - \delta_k I)X_{k+1} &= X_k(B - \delta_k I) + C, \\ X_{k+2}(B - \tau_k I) &= (A - \tau_k I)X_{k+1} - C,\end{aligned}$$

for  $k = 0, 1, 2, \dots$ . Here  $X_0$  is a given initial approximate solution, and the  $\delta_k$  and  $\tau_k$  are real or complex parameters chosen so that the computed approximate solutions  $X_k$  converge rapidly to the solution  $\tilde{X}$  of the Sylvester equation as  $k$  increases. This paper discusses the possibility of solving one of the equations in the ADI iterative method more often than the other one, i.e., relaxing the strict alternation requirement, in order to achieve a higher rate of convergence. Our analysis based on potential theory shows that this generalization of the ADI iteration method can give faster convergence than when strict alternation is required.