ICM-9209-41 Gaussian Quadrature Applied to Adaptive Chebyshev Iteration, D.Calvetti, G.H. Golub and L. Reichel.

ABSTRACT: Chebyshev iteration has been a popular iterative scheme for the solution of large linear systems of equations with a symmetric positive definite matrix A. With the advent of parallel processors, there has been a resurgence of interest in this method. In Chebyshev iteration one determines iteration parameters so that the residual polynomials are scaled Chebyshev polynomials for some interval $[a, b]$ on the positive real axis. Chebyshev iteration is often implemented as an adaptive iteration scheme, in which one during the iterations seeks to determine an interval $[a, b]$ that make the iterates converge rapidly to the solution of the linear system. Roughly, the interval should contain most of the spectrum of A and be as small as possible. Recently, Golub and Kent [6] proposed a new adaptive Chebyshev iteration method, in which inner products of residual vectors are interpreted as modified moments. These modified moments and the recursion coefficients for the residual polynomials yield a symmetric tridiagonal matrix, whose eigenvalues are used to determine an interval $[a, b]$. The eigenvalues are nodes of a Gaussian quadrature rule. We propose a modification of this scheme, in which the determination of a suitable interval is based on the weights of this quadrature rule also. Computer examples illustrate that a significant reduction in the number of iterations can be achieved by this modification.