ICM-9301-45 Analysis of a Simplified Liquid Crystal Shear-Flow Model, E.C. Gartland, Jr., SIAM Journal on Applied Mathematics (to appear).

ABSTRACT This paper is concerned with the analysis of the nonlinear two-point boundary value problem $\phi'' = \lambda\, x \cos^2 \phi$, $\phi(-1/2) = 0$, $\phi(1/2) = m\pi$. This system models the preferred direction of orientation of liquid crystal molecules flowing in a channel. It is derived by simplification of a formulation in terms of the Ericksen-Leslie equations. The problem has interesting bifurcation and singular-perturbation phenomena. There are infinitely many distinct solution branches, the number growing roughly quadratically with the parameter λ, and individual solutions typically possess $O(1/\sqrt{\lambda})$ boundary layers at each end point and an $O(1/\sqrt[3]{\lambda})$ interior layer at $x = 0$. Analytical techniques (involving differential inequalities of Nagumo type) can be used to rigorously prove the existence of the infinite family of stable solutions; while numerical investigations (using the COLSYS/COLNEW and AUTO86 packages) are used to explore the unstable solutions and bifurcation behavior, which involves limit/turning points and simple bifurcation points.