

ICM-9302-48 Optimal Semi-iterative Methods Applied to SOR in the Mixed Case, M. Eiermann and R.S. Varga.

ABSTRACT The application of optimal semi-iterative methods to the standard successive over-relaxation (SOR) iterative method, with any real relaxation parameter ω , is completely analyzed here, under the assumptions that the associated Jacobi matrix B is consistently ordered and weakly cyclic of index 2 and that the spectrum, $\sigma(B^2)$, of B^2 satisfies $\sigma(B^2) \subset [-\alpha^2, \beta^2]$ with $0 < \alpha < \infty$ and $0 < \beta < 1$. The spectrum of B^2 is then a mixture of positive and negative eigenvalues, the so-called “mixed case”. If $\kappa(\Omega_{\omega, \alpha, \beta})$ denotes the optimal asymptotic convergence factor for semi-iteration applied to \mathcal{L}_ω (the associated SOR iteration matrix), we deduce that

$$1 > \min_{\omega \in \mathbb{R}} \rho(\mathcal{L}_\omega) > \min_{\omega \in \mathbb{R}} \kappa(\Omega_{\omega, \alpha, \beta}) = [(\sqrt{1 + \alpha^2} - \sqrt{1 - \beta^2})^2] / [\alpha^2 + \beta^2].$$