ABSTRACT The application of optimal semi-iterative methods to the standard successive over-relaxation (SOR) iterative method, with any real relaxation parameter $\omega$, is completely analyzed here, under the assumptions that the associated Jacobi matrix $B$ is consistently ordered and weakly cyclic of index 2 and that the spectrum, $\sigma(B^2)$, of $B^2$ satisfies $\sigma(B^2) \subseteq [-\alpha^2, \beta^2]$ with $0 < \alpha < \infty$ and $0 < \beta < 1$. The spectrum of $B^2$ is then a mixture of positive and negative eigenvalues, the so-called “mixed case”. If $\kappa(B_{\omega, \alpha, \beta})$ denotes the optimal asymptotic convergence factor for semi-iteration applied to $L_\omega$ (the associated SOR iteration matrix), we deduce that

$$1 > \min_{\omega \in \mathbb{R}} \rho(L_\omega) > \min_{\omega \in \mathbb{R}} \kappa(B_{\omega, \alpha, \beta}) = \left[(\sqrt{1 + \alpha^2} - \sqrt{1 - \beta^2})^2 / (\alpha^2 + \beta^2)\right].$$