

ICM-9302-49 An Algorithm for Determining if the Inverse of a Strictly Diagonally Dominant Stieltjes Matrix is Strictly Ultrametric, Richard S. Varga and Reinhard Nabben.

ABSTRACT It was recently shown that the inverse of a strictly ultrametric matrix is a strictly diagonally dominant Stieltjes matrix. On the other hand, as it is well-known that the inverse of a strictly diagonally dominant Stieltjes matrix is a real symmetric matrix with nonnegative entries, it is natural to ask, conversely, if every strictly diagonally dominant Stieltjes matrix has a strictly ultrametric inverse. Examples show, however, that the converse is not true in general, i.e., there are strictly diagonally dominant Stieltjes matrices in $\mathbb{R}^{n \times n}$ (for every $n \geq 3$) whose inverses are not strictly ultrametric matrices. Then, the question naturally arises if one can determine which strictly diagonally dominant Stieltjes matrices, in $\mathbb{R}^{n \times n}$ ($n \geq 3$), have inverses which are strictly ultrametric. Here, we develop an algorithm, based on graph theory, which determines if a given strictly diagonally dominant Stieltjes matrix A has a strictly ultrametric inverse, where the algorithm is applied to A and requires no computation of inverse. Moreover, if this given strictly diagonally dominant Stieltjes matrix has a strictly ultrametric inverse, our algorithm uniquely determines this inverse as a special sum of rank-one matrices.