

Lehmer Pairs of Zeros and the Riemann ξ -Function

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ABSTRACT

In this note, a rigorous formulation is given for a pair of consecutive simple positive zeros, of the Riemann ξ -function, to be a “Lehmer pair” of zeros. It is then shown that such Lehmer pairs of zeros do exist, and that each such pair gives a lower bound for the de Bruijn-Newman constant Λ (where the Riemann Hypothesis is equivalent to $\Lambda \leq 0$). As a constructive application of this theory, the following new lower bound for Λ is obtained:

$$-4.379 \cdot 10^{-6} < \Lambda.$$

The theoretical contribution of this note is a proof (based on the result of Littlewood) that if infinitely many Lehmer pairs of zeros exist, then $0 \leq \Lambda$, which is an improved version of Corollary 1 of Csordas, Smith, and Varga [CSV].