Generalized Ultrametric Matrices -
a Class of Inverse M-Matrices

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ABSTRACT

Recently, Martínez, Michon, and San Martín [8] introduced the new class of (symmetric) 
strictly ultrametric matrices. They proved that the inverse of a strictly ultrametric matrix is a 
strictly row and strictly column diagonal dominant Stieltjes matrix. Here, we generalize the 
result of [8] by introducing a class of nonsymmetric matrices, called generalized ultrametric ma-
trices. We give a necessary and sufficient condition for the regularity of these matrices and prove 
that the inverse of a nonsingular generalized ultrametric matrix is a row and column diagonally 
dominant M-matrix. We establish that a nonnegative matrix is a generalized ultrametric matrix 
if and only if the matrix is a certain sum of at most rank-two matrices. Moreover, we give a 
characterization of generalized ultrametric matrices, based on weighted trees. The entries of gen-
eralized ultrametric matrices then arise as certain “distances” between the leaves and the root of 
the tree.