

Generalized Ultrametric Matrices -
a Class of Inverse M–Matrices

Reinhard Nabben
Fakultät für Mathematik
Universität Bielefeld
Postfach 10 01 31
33 501 Bielefeld
Germany

and

Richard S. Varga
Institute for Computational Mathematics
Kent State University
Kent, OH 44242-0001 USA

ABSTRACT

Recently, Martínez, Michon, and San Martín [8] introduced the new class of (symmetric) *strictly ultrametric matrices*. They proved that the inverse of a strictly ultrametric matrix is a strictly row and strictly column diagonally dominant Stieltjes matrix. Here, we generalize the result of [8] by introducing a class of nonsymmetric matrices, called *generalized ultrametric matrices*. We give a necessary and sufficient condition for the regularity of these matrices and prove that the inverse of a nonsingular generalized ultrametric matrix is a row and column diagonally dominant M–matrix. We establish that a nonnegative matrix is a generalized ultrametric matrix if and only if the matrix is a certain sum of at most rank-two matrices. Moreover, we give a characterization of generalized ultrametric matrices, based on weighted trees. The entries of generalized ultrametric matrices then arise as certain “distances” between the leaves and the root of the tree.