

A Sufficient Condition for every Class
of Inverse Z -Matrices

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ABSTRACT

Recently, a classification of matrices of class Z was introduced by Fiedler and Markham. This classification contains the classes of M -matrices and the classes of N_0 and F_0 -matrices studied by K. Fan, G. Johnson, and R. Smith. The problem of determining which nonsingular matrices have inverses which are Z -matrices is called the *inverse Z -matrix problem*. For special classes of Z -matrices, such as the M -matrices and N_0 -matrices, there exist at least partial results, i.e. special classes of matrices were introduced for which the inverse of such a matrix is an M -matrix or an N_0 -matrix. Here, we define a system of classes of matrices for which the inverse of each matrix of each class belongs to one class of the classification of Z -matrices defined by Fiedler and Markham. Moreover, certain properties of the matrices of each class are established, e.e., inequalities for the sum of the entries of the inverse and the structure of certain Schur complements. We also give a necessary and sufficient condition for regularity. The class of inverse N_0 -matrices given here generalizes the class of inverse N_0 -matrices discussed by R. Smith. All results established here can be applied to a class of distance matrices which corresponds to a non-archimedean metric. This metric arises in the p -adic number theory and in taxonomy.