Taking into account preliminary information in the EPH

Bernard Beauzamy
August 6th, 2007

At present, the EPH is built using only precise information: we know that some measure gave some value, and this precise value propagates as a probabilistic law inside the EPH.

One may want to use as initial information a non-deterministic information, that is an information which is already of probabilistic nature. This may happen for at least two reasons:

The measure itself has some error, which is usually given by a probabilistic law (the error upon the measure is not taken into account in the present construction of the EPH).

Some expert knowledge may be used, under some vague form: for instance, the result at this point should be between this value and that value. This could be described by a uniform law on some interval (or by another type of law, if the expert gives it).

The purpose of this note is to show how to incorporate such an information.

Simple case: two values

Assume first, for simplicity, that at the point $x_0$, we have two possible values $t_1$ and $t_2$, each of them with probability 1/2.

At any point $x$ of the EPH, the value $t_1$ would lead to a density which we denote by $f(t; t_1)$ (the points $x$ and $x_0$ are omitted in the notation). This is simply the standard construction of the EPH, assuming we found $t_1$ at $x_0$.

The same way, if we found $t_2$ at $x_0$, this would lead to a density $f(t; t_2)$ at any point $x$.

Since we have either $t_1$ or $t_2$, with probability 1/2 each, the density $f(t)$ at a point $x$ is:

$$ f(t) = \frac{1}{2} \left( f(t; t_1) + f(t; t_2) \right) $$
General discrete case

Assume now that, at the point $x_0$, we have any number of possible results $t_i$, with respective probabilities $p_i$, $\sum p_i = 1$. Then, at any point $x$ of the EPH, the propagated density function will be:

$$f(t) = \sum_i p_i f(t ; t_i)$$

General continuous case

Assume that, at the point $x_0$, we have a density $h(\tau)$ of possible temperatures (in general, $\tau \in ]-\infty, +\infty[$). Then the density at any point of the EPH will be given by the formula:

$$f(t) = \int_{-\infty}^{+\infty} f(t ; \tau) h(\tau) d\tau$$

The densities indicated by the above formulas are the influence (at the point $x$) coming from the information at the point $x_0$. If now we have several points $x_0$, we need to combine the influences, at a given point $x$, coming from all source points: this is done as in the previous construction of the EPH.