

MathML: the inside story

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Basics of MathML

Presentation Tags

Example

$$a^2 + \frac{b}{2} // \text{MathMLForm}$$

```
<math>
<mrow>
  <msup>
    <mi>a</mi>
    <mn>2</mn>
  </msup>
  <mo>+</mo>
  <mfrac>
    <mi>b</mi>
    <mn>2</mn>
  </mfrac>
</mrow>
</math>
```

Tags

Roughly the same as in Mathematica, TeX, ... except tokens must be tagged.

Token Elements

mi	identifier
mn	number
mo	operator, fence, or separator
mtext	text
mspace	space
ms	string literal
mglyph	adding new character glyphs to MathML

General Layout Schemata

mrow	group any number of sub-expressions horizontally
mfrac	form a fraction from two sub-expressions
msqrt	form a square root (radical without an index)
mroot	form a radical with specified index
mstyle	style change
merror	enclose a syntax error message from a preprocessor
mpadded	adjust space around content
mphantom	make content invisible but preserve its size
mfenced	surround content with a pair of fences
menclose	enclose content with a stretching symbol such as a long division sign.

Script and Limit Schemata

msub	attach a subscript to a base
msup	attach a superscript to a base
msubsup	attach a subscript-superscript pair to a base
munder	attach an underscript to a base
mover	attach an overscript to a base
munderover	attach an underscript-overscript pair to a base
mmultiscripts	attach prescripts and tensor indices to a base

Tables and Matrices

mtable	table or matrix
mlabeledtr	row in a table or matrix with a label or equation number
mtr	row in a table or matrix
mtd	one entry in a table or matrix

maligngroup alignment markers
malignmark alignment markers

Not good at elementary school math (long division, ...) which is typically tabular

Content Tags

Example

$$a/2 + b^2$$

```
<math>
<apply><plus/>
  <apply><divide/>
    <ci>a</ci>
    <cn>2</cn>
  </apply>
  <apply><power/>
    <ci>b</ci>
    <cn>2</cn>
  </apply>
</apply>
</math>
```

Tags

About 150 tags roughly covering up to 2nd year math in college.

Content tag list from [MathML spec](#)

Like presentation MathML tags, content MathML tags are not good at elementary school math (long division, ...) which is typically tabular

Glue

<math>

top-level tag that wraps all MathML

<semantics>

Associates additional representations with some MathML.

$\text{T}_{\text{E}}\text{X}$, *Mathematica*, OpenMath

<maction>

Provides a way of specifying some interactive action such as:

- toggling between expressions
- show something on the status line
- having a tooltip
- highlighting
- popup menu

MathML renderers are not required to support any of these — they really only make sense in a document interface.

History of MathML

MathML went from being a W3C committee to a W3C recommendation in less than two years.

Subsequent work has been polishing the original recommendation and keeping it up to date with the ever changing Web.

Milestones

- 1994 HTML 3.0 working draft includes a proposal for Math
- Apr, 1995 WWW Conference in Darmstadt panel session on Math
- Nov, 1995 Wolfram Research presents proposal to W3C for Math in HTML
- May, 1996 Digital Library Initiative meeting in Champaign–Urbanabring together interested parties.
HTML Math Editorial Review Board was formed soon afterwards
- Mar, 1997 Formally reconstituted as the first W3C Math Working Group
- Apr, 1998 MathML W3C recommendation published
- Jul, 1999 MathML 1.01 revision
- Feb, 2001 MathML 2.0 W3C recommendation published

MathML group has been rechartered to last until May, 2003.
See <http://www.w3.org/Math/Documents/Charter2001.html> for more info.

Work with other groups

Work with STIX/STIPUB to add 600 new mathematical characters to Unicode.
These will be in Unicode 3.2 (Fall 2001)

Work with the OpenMath group so that MathML and OpenMath don't conflict.

Design Decisions

Structured Presentation

From the earliest days, the MathML working drafts included structured presentation. This was not without controversy...

- some members only wanted "content" MathML
- they did concede content-only MathML would limit MathML's expansibility
- some members were very opposed to disambiguation characters (see below)

- Structured Presentation aids in line breaking and indentation

$$\int x \operatorname{Sin}[x - a]^n dx$$

$$a \cos(a - x) {}_2F_1\left(\frac{1}{2}, \frac{1 - n}{2}; \frac{3}{2}; \cos^2(a - x)\right) (-\sin(a - x))^n \sin(a - x) \sin^2(a - x)^{\frac{1}{2}(-n-1)} +$$

$$\frac{1}{2} (-\sin(a - x))^n \sin(a - x) \left(\frac{2(a - x) \cos(a - x) {}_2F_1\left(1, \frac{n}{2} + 1; \frac{n+3}{2}; \sin^2(a - x)\right)}{n + 1} - \right.$$

$$\left. 2^{-n-1} \sqrt{\pi} \Gamma(n + 1) {}_3\tilde{F}_2\left(1, \frac{n}{2} + 1, \frac{n}{2} + 1; \frac{n+3}{2}, \frac{n}{2} + 2; \sin^2(a - x)\right) \sin(a - x) \right)$$

- Structured Presentation aids accessibility
- All W3C proposals are reviewed to make sure they meet accessibility guidelines.

$t(x + y)$ — "t of x plus y" vs. "t times quantity x plus y"

```
<math xmlns="http://www.w3.org/1998/Math/MathML">
<mrow>
```

```

<mi>t</mi>
<mo>&ApplyFunction;</mo>
<mrow>
<mo>( </mo>
  <mrow>
    <mi>x</mi>
    <mo>+</mo>
    <mi>y</mi>
  </mrow>
  <mo>)</mo>
</mrow>
</mrow>
</math>

```

Characters that resolve ambiguity (will be part of Unicode 3.2):

⁡
 ⁢
 ⁣
 ⅆ
 ⅇ
 ⅈ

Structure is useful for adding pauses.

- Structured Presentation aids in selection

$$\begin{aligned}
 &a^{22} + 22 a^{21} b + 231 a^{20} b^2 + 1540 a^{19} b^3 + 7315 a^{18} b^4 + 26334 a^{17} b^5 + \\
 &74613 a^{16} b^6 + 170544 a^{15} b^7 + 319770 a^{14} b^8 + 497420 a^{13} b^9 + 646646 a^{12} b^{10} + \\
 &705432 a^{11} b^{11} + 646646 a^{10} b^{12} + 497420 a^9 b^{13} + 319770 a^8 b^{14} + 170544 a^7 b^{15} + \\
 &74613 a^6 b^{16} + 26334 a^5 b^{17} + 7315 a^4 b^{18} + 1540 a^3 b^{19} + 231 a^2 b^{20} + 22 a b^{21} + b^{22}
 \end{aligned}$$

- Structured Presentation aids in computation (parsing)
 No parsing issues since expression is fully parsed — just need to map character (e.g., \leq) to the corresponding function.
 Hence, for common notations, presentation and content MathML are equally easy to interpret for computation systems.

Syntax

The biggest arguments concerned syntax.

Human authorable MathML was one of the goals listed in the MathML charter. Many people felt human authorability was one of the reasons HTML was so successful.

A number of different "input" syntaxes were proposed, spanning the spectrum from a powerful augmented operator precedence grammar to a very trivial grammar that only contained a few common operators such as "+" and "*".

As the arguments continued, the XML working draft seemed to be gaining more and more favor and everyone agreed that any grammar agreed upon for MathML should allow XML as a subset.

Ultimately, the MathML committee couldn't reach agreement on a input syntax and decided that the marketplace should on the syntax. However, for compatibility, all MathML readers would need to accept the XML syntax and filters would translate between various input syntaxes and the XML syntax. The various input syntaxes could be part of a *semantics* element and coexist in a document.

Agreeing to use XML was quite visionary. The decision to use XML was made a year before XML 1.0 became a W3C recommendation in Feb, 1998. MathML became a recommendation in April, 1998 and was the first XML application.

XML Syntax is quite verbose.

The largest initial criticism of MathML was because of the XML syntax.

Are you guys nuts!

Expand [$(x + y)^{100}$]

$$\begin{aligned}
 &x^{100} + 100 y x^{99} + 4950 y^2 x^{98} + 161700 y^3 x^{97} + 3921225 y^4 x^{96} + 75287520 y^5 x^{95} + \\
 &1192052400 y^6 x^{94} + 16007560800 y^7 x^{93} + 186087894300 y^8 x^{92} + 1902231808400 y^9 x^{91} + \\
 &17310309456440 y^{10} x^{90} + 141629804643600 y^{11} x^{89} + 1050421051106700 y^{12} x^{88} +
 \end{aligned}$$

$$\begin{aligned} &7110542499799200 y^{13} x^{87} + 44186942677323600 y^{14} x^{86} + 253338471349988640 y^{15} x^{85} + \\ &1345860629046814650 y^{16} x^{84} + 6650134872937201800 y^{17} x^{83} + 30664510802988208300 y^{18} x^{82} + \\ &132341572939212267400 y^{19} x^{81} + 535983370403809682970 y^{20} x^{80} + 2041841411062132125600 y^{21} x^{79} + \\ &7332066885177656269200 y^{22} x^{78} + 24865270306254660391200 y^{23} x^{77} + \\ &79776075565900368755100 y^{24} x^{76} + 242519269720337121015504 y^{25} x^{75} + \\ &699574816500972464467800 y^{26} x^{74} + 1917353200780443050763600 y^{27} x^{73} + \\ &4998813702034726525205100 y^{28} x^{72} + 12410847811948286545336800 y^{29} x^{71} + \\ &29372339821610944823963760 y^{30} x^{70} + 66324638306863423796047200 y^{31} x^{69} + \\ &143012501349174257560226775 y^{32} x^{68} + 294692427022540894366527900 y^{33} x^{67} + \\ &580717429720889409486981450 y^{34} x^{66} + 1095067153187962886461165020 y^{35} x^{65} + \\ &1977204582144932989443770175 y^{36} x^{64} + 3420029547493938143902737600 y^{37} x^{63} + \\ &5670048986634686922786117600 y^{38} x^{62} + 9013924030034630492634340800 y^{39} x^{61} + \\ &13746234145802811501267369720 y^{40} x^{60} + 20116440213369968050635175200 y^{41} x^{59} + \\ &28258808871162574166368460400 y^{42} x^{58} + 38116532895986727945334202400 y^{43} x^{57} + \\ &49378235797073715747364762200 y^{44} x^{56} + 61448471214136179596720592960 y^{45} x^{55} + \\ &73470998190814997343905056800 y^{46} x^{54} + 84413487283064039501507937600 y^{47} x^{53} + \\ &93206558875049876949581681100 y^{48} x^{52} + 98913082887808032681188722800 y^{49} x^{51} + \\ &100891344545564193334812497256 y^{50} x^{50} + 98913082887808032681188722800 y^{51} x^{49} + \\ &93206558875049876949581681100 y^{52} x^{48} + 84413487283064039501507937600 y^{53} x^{47} + \\ &73470998190814997343905056800 y^{54} x^{46} + 61448471214136179596720592960 y^{55} x^{45} + \\ &49378235797073715747364762200 y^{56} x^{44} + 38116532895986727945334202400 y^{57} x^{43} + \\ &28258808871162574166368460400 y^{58} x^{42} + 20116440213369968050635175200 y^{59} x^{41} + \\ &13746234145802811501267369720 y^{60} x^{40} + 9013924030034630492634340800 y^{61} x^{39} + \\ &5670048986634686922786117600 y^{62} x^{38} + 3420029547493938143902737600 y^{63} x^{37} + \\ &1977204582144932989443770175 y^{64} x^{36} + 1095067153187962886461165020 y^{65} x^{35} + \\ &580717429720889409486981450 y^{66} x^{34} + 294692427022540894366527900 y^{67} x^{33} + \\ &143012501349174257560226775 y^{68} x^{32} + 66324638306863423796047200 y^{69} x^{31} + \\ &29372339821610944823963760 y^{70} x^{30} + 12410847811948286545336800 y^{71} x^{29} + \end{aligned}$$

$$\begin{aligned}
&4998813702034726525205100 y^{72} x^{28} + 1917353200780443050763600 y^{73} x^{27} + \\
&699574816500972464467800 y^{74} x^{26} + 242519269720337121015504 y^{75} x^{25} + \\
&79776075565900368755100 y^{76} x^{24} + 24865270306254660391200 y^{77} x^{23} + \\
&7332066885177656269200 y^{78} x^{22} + 2041841411062132125600 y^{79} x^{21} + 535983370403809682970 y^{80} x^{20} + \\
&132341572939212267400 y^{81} x^{19} + 30664510802988208300 y^{82} x^{18} + 6650134872937201800 y^{83} x^{17} + \\
&1345860629046814650 y^{84} x^{16} + 253338471349988640 y^{85} x^{15} + 44186942677323600 y^{86} x^{14} + \\
&7110542499799200 y^{87} x^{13} + 1050421051106700 y^{88} x^{12} + 141629804643600 y^{89} x^{11} + \\
&17310309456440 y^{90} x^{10} + 1902231808400 y^{91} x^9 + 186087894300 y^{92} x^8 + 16007560800 y^{93} x^7 + \\
&1192052400 y^{94} x^6 + 75287520 y^{95} x^5 + 3921225 y^{96} x^4 + 161700 y^{97} x^3 + 4950 y^{98} x^2 + 100 y^{99} x + y^{100}
\end{aligned}$$

MathML compresses well.

Bytes Used (kb):

	Normal	Compressed
<i>Mathematica</i>	3.5	1.2
MathML	39.6	1.8

Many people asked why didn't we use T_EX?

Which part of T_EX?

T_EX is not amenable to the growing number of XML tools such as CSS, XSLT, DOM, parsers...

Content MathML

Not all members were in favor of it for MathML 1.

Original content proposal used <EXPR> as a grouping tag and used an infix notation.

Eg, $ax + b$ might be encoded as:

```
<EXPR> <MI>a</MI> <TIMES/> <MI>x</MI> <PLUS/> <MI>b</MI> </EXPR>
```

Notice that grouping of the ax was not required.

Many of the people proposing content were opposed to presentation and the earlier form allowed specifying some things not possible in the latter notation.

A late change in syntax to the present regular lisp-likeform delayed MathML a few months.

Due to greater emphasis and discussion on presentation MathML, content MathML was not as well thought out as Presentation MathML. As evidence of this, MathML2 has many content fixes (deprecates `<fn>` and `<reln>`), and adds some tags that were glaring omissions (eg, `<lcm/>`).

Another look at MathML

MathML is mainly about publishing...

What Content MathML is really about

Content MathML is *not* really designed for computation

MathML purposely does not contain an "evaluate" token.

What a MathML application should do when it receives the following is not defined

```
<apply> <sin/>
  <apply><divide/>
    <cn type="constant"> &pi; </cn>
    <cn>4</cn>
  </apply>
</apply>
```

A computation system *might* display (or return) any of the following:

$$\sin\left(\frac{\pi}{4}\right)$$

$$\sin(\pi/4)$$

$$\frac{1}{\sqrt{2}}$$

$$1/\sqrt{2}$$

$$0.707107$$

$$0.707106781186547524400844362105$$

$$\text{Sin}[\pi/4]$$

Content MathML is really designed for publishing

The main use of MathML (so far) is for documentation.
Content MathML abstracts the notation away from the representation.

Various publishers (or readers) have their preferred notation:

$\sin(x)$	$\sin x$	$\text{Sin}[x]$
-----------	----------	-----------------

$\frac{1}{2} x$	$\frac{x}{2}$	$x/2$
-----------------	---------------	-------

$\int_0^\pi \sqrt{x^2 - 1} dx$	$\int_0^\pi \sqrt{x^2 - 1} dx$
--------------------------------	--------------------------------

Content MathML is closer to the intent of XML than Presentation MathML

Content MathML abstracts the logical meaning of an expression from its notational display.

Style sheets or other means of rendering decide on what it should look like in much the same way as style sheets are used to decide the font size, font, indentation, and alignment of a title or section heading.

What Presentation MathML is really about

Presentation MathML exists for some very important reasons even though it specifies layout much more strongly than does content MathML.

- There are a small number of common notational structures whereas there are a very large number of common functions/concepts. New notational structures are introduced much less frequently than new mathematical concepts.
- Many mathematical authors are very picky about notation.

Unlike text, notation conveys a great deal of meaning. Eg, a x has a different meaning from a x in most mathematical texts. This is not true of most prose.

- Notations can also be used to convey important ideas such as cancellation in division.

$$762. \frac{\text{mile}}{\text{hour}} \frac{\text{hour}}{3600 \text{ sec}} \frac{5280 \text{ feet}}{\text{mile}}$$

762. miles / hour hour / (3600 sec) 5280 feet / mile

- The stylistic capabilities of most text-based systems can at best produce mediocre quality math. Presentation MathML serves as the low level formatting commands to which content MathML can be converted.

CSS is the main stylistic engine of Web browsers today. It defines a set of stylistic changes that all Web-based renderers are supposed to support. The MathML Working Group is working on getting some of the things used for presentation MathML into CSS so that stylistic settings can be used to render math.

A similar technology is XSL FO (formatting/flow objects). These would define the low level controls that XSL transformations could transform into. The W3C XSL FO committee has agreed to using presentation MathML (tags and attributes) as its Math component.

Status of MathML

Lots and lots of interest and some places using it for real

- Used for archival format
U.S. patent office publishes 3,000–4,000 patents/week which have 1,500 math items per week
SIGSAM Bulletin
- Browser implementations and browser "plug-ins"
[techexplorer](#) (IBM)
WebEQ/MathPlayer (Design Science)
[Amaya](#)
[Mozilla](#)
- Commercial products
Mathematica
Maple
MathType
...
- T_EX
Several T_EX to MathML translators available (all still beta)
MathML to T_EX translators available (MathML spec's postscript version was produced this way)
- Research efforts
- Distance learning
WebCT @ University of Georgia
papers later today...
- Conformance test suite
(full of surprises)

Reasons for success

- Fills a big hole/need
All of the other methods for putting math on the web are poor
- Backing of the major players
There are no other serious proposals
- (Mostly) used well proven design
cover the basics and pick a modest goal and get it done
- Run with the crowd
MathML leverages off of all of the web-related tools.
- Luck

Once again, computer math is on the cutting edge...

Problems with Plug-ins/Applets/ActiveX

Applets and plug-in design and implementations were not well thought out. They basically treat the contents as a presized static picture.

- Width/height of plug-in/applet must be set by writer, not renderer.
- Lack of environmental information.
Plug-ins and applets can't query the current font size, background color, drawing width.
- Baseline
Can't align the baseline of math with the surrounding text.

- Printing

If printing worked, it was not high resolution

Lots of applets/page is a problem

Ron Whitney gathered some statistics from a 26 page AMS Journal paper on the size and amount of math in the paper (authored in T_EX).

1268 pieces of math, 474 different — 49 pieces of math/page

# atoms	occurrences
≤1	517 (41%)
≤2	684 (54%)
≤3	799 (63%)
≤9	1100 (87%)

Longest was 168 atoms.

Characters and Fonts

Out of the box, most machines have only Symbol font for math.
Symbol font has ≈120 chars, about 40 of which are Greek letters.

Mathematica has about 700 mathematical characters in its fonts, and unicode 3.2 adds another 400.

Writing installers for fonts is not easy...

Mozilla doesn't include MathML in its nightly builds even though Design Science has contributed fonts to Mozilla because no one has written an installer for the fonts (for all platforms).

Standards

MathML has acted as a test case for many W3C standards

CSS — baseline alignment got added

DOM — how to interface with application-specific tree models

MIME types — how to associate application specific XML renderers with tags

IE and Mozilla

Although there are two good "plug-ins" for MathML, neither Netscape nor IE (<=5) have good support for the features they need.

IE 6

IE 5.5 fixes many of the problems with the ActiveX interface; IE 6 is good.

IBM's techexplorer and Design Science's WebEq (MathPlayer) sent lots of bug reports in.

Mozilla

Mozilla has built-in support for MathML.

Downside is that you are stuck with whatever quality the Mozilla volunteers implement.

The Future of MathML

Users

The user community drives features.

Awaiting the next round of browsers for mass adoption of MathML.

Fonts may still be a hang up.

Work with other standards groups to make sure the standards are MathML–friendly.
Eg, CSS and XML FO MathML support.

Some Presentation Featurettes

Better grade school math support (2–Daddition and multiplication, long division)

$$\begin{array}{r} 12 \\ \times 34 \\ \hline 48 \\ 36 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 10 \\ 131 \overline{) 1413} \\ \underline{131} \\ 103 \end{array}$$

In MathML, the long division example would look like:

```
<table columnspacing='0' rowspacing='0'>
<tr>
<td></td>
<td columnalign='right'><mn>10</mn></td>
</tr>
<tr>
<td columnalign='right'><mn>131</mn></td>
<td columnalign='right'>
<math notation='longdiv'><mn>1413</mn></math>
</td>
</tr>
<tr>
<td></td>
<td columnalign='right'>
<math>
<math>131</math>
<math>&UnderBar; </math>
<math>3</math>
</math>
</td>
</tr>
<tr>
<td></td>
<td columnalign='right'><mn>103</mn></td>
</tr>
</table>
```

Grade school notations are not as universally adopted as more advanced notations

Ability to nest other XML inside of MathML

Might be useful to allow SVG or other graphics for tokens

Might be useful to allow font changes in a token: **log** vs. log

Lots more content tags

Content tags are "cheap".

Functions that have multiple notations are prime candidates

Continued fractions

Binomially, Multinomial, Gamma, Jacobi symbol...

Unit step, Dirac delta

Simple functions that lack common notation

round

fractional part

sign

Functions for completeness

MathML has several vector/matrix functions, but not Eigenvalues and Eigenvectors

MathML has several set functions, but not Complement

Some evaluation related tags

Some evaluation related tags might get added in the future.

Web services are a hot topic; some people feel their should be a standard for math related web service.

Tags listed below would differ from the other MathML tags since their support would be optional — web browsers and other renderers would probably not support them.

Two points to remember about computation

- MathML content is currently aimed at display, not computation.
This is what the vast majority of users are interested in.
- Presentation and content are the same for most simple math, so either could be used for evaluation.

Add an *evaluate* tag

For some systems that support MathML, evaluation is implied.

E.g., a computation system that accepts MathML as input and generates MathML as output.

If an *evaluate* tag is added, then the semantics would probably be vaguely defined just as what MathML means today to different systems is vaguely defined. More precise specification raises all sorts of issues that probably are not easily supported by current systems.



$\sin(\pi / 4)$
 $\frac{1}{\sqrt{2}}$
0.707107
0.707106781186547524400844362105

If *evaluate* is added, there will probably be a way to distinguish between numerical and symbolic evaluation.

Add assignment

If assignment is added, then there is an issue about state: when does one *math* tag inherit the state of a previous *math* tag's evaluation?

Add Control flow

- branches
issues of structural equality, numerical equality, how much effort is a system required to do to try to determine the equality.
- loops
other than issues of evaluation and assignment semantics, these are simple

Add Commands

The current tags represent Mathematical objects, not commands.

If you want to do computation, there are many common commands that people use. E.g.

Solve

Fit

Expand

Simplify

Plot

What these commands do are much more system-oriented than others, so perhaps they don't belong in MathML and should really be part of the wrapper as in [webMathematica](#).

Shameless advertisement

- For ten years, *Mathematica* has been able to have evaluations run on remote kernels. Multiple kernels can be used from one Frontend.
- With techexplorer, you can save a notebook to a format that can be read by a browser and still be re-evaluated (remotely) if desired.
- With *webMathematica*, you can set up computation servers, with all of the security, load balancing, etc., that apache servers provide. You can write servlets etc., in Java or *Mathematica*.

The High Court of History

There has been about five years of work put into MathML.
Only time will tell whether it has been a service to the community or not.

When at some future date the high court of history sits in judgment on each one of us—recording whether in our brief span of service we fulfilled our responsibilities to the state—our success or failure, in whatever office we may hold, will be measured by the answers to four questions—were we truly men of courage ... were we truly men of judgment ... were we truly men of integrity ... were we truly men of dedication?

John F. Kennedy — Jan, 1961