Flexible Mathematical GUI Controls for Mathematical Electronical Documents

Ralf Hillebrand
MuPAD Research Group
University of Paderborn
Department of Mathematics and Computer Science

tonner@mupad.de

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Abstract
In cooperation with SciFace Software\textsuperscript{1} the MuPAD Group in Paderborn develops prototypes for open and flexible mathematical user interfaces which can be integrated into open software environments such as presentation and authoring tools as well as Internet browsers.

The main goal of our project is to provide authors of interactive educational documents with a variety of flexible tools which ease the author's burden of dealing with the technical issues of the software environment.

1 Introduction

In the last years electronical tools entered the education of mathematics in different ways. In addition to graphical pocket calculators, computer algebra systems such as Derive, Maple, Mathematica or MuPAD as well as special mathematical software like Euclid or Mathview are nowadays used in schools and universities for educational purposes.

Presentation tools such as MS PowerPoint, Internet Browsers or professional authoring tools such as Toolbook\textsuperscript{2}, Director or Authorware\textsuperscript{3} are presently used for writing educational mathematical documents.

We develop software components which can be used as plugins in a wide variety of "container software". This paper will emphasize the usability of these tools for Web documents but please keep in mind that they can also be embedded in word processing programs like MS Word, presentation tools like

\textsuperscript{1}SciFace Software GmbH & Co. K.G, Paderborn, Germany, \url{http://www.sciface.com/}.
\textsuperscript{2}Toolbook is a product of Asymetric Inc.; \url{http://www.asymetrix.com/}.
\textsuperscript{3}Director and Authorware are products of Macromedia Inc.; \url{http://www.macromedia.com/}.
MS PowerPoint and professional authoring tools like Toolbook, Authorware or Director mentioned above. This enables an author of interactive mathematical electronical documents to use the presentation software of his choice and to employ software components for sophisticated mathematical tasks.

2 The Mathematical GUI Controls

Presently Windows 95/97/NT are becoming more and more important in schools and universities. The ActiveX technology under Windows 95/98/NT [Ar97] provides a technical concept for the development of flexible controls which can be integrated into the MS Internet Explorer or into other software packages. Each of these Controls defines a set of interface functions which can be used from outside to interact with it. For example, a Control for visualization may offer a function for drawing a graph of a given function, or functions for rotating or zooming a displayed graph.

The author of a Web page uses JavaScript to access the interface functions and to implement the communication between serveral Controls.

Using this powerful technology we are developing three ActiveX Controls under Windows 95/98/NT:

1. A Editor Control for input and output of mathematical expressions.

2. A Calculator Control for mathematical computations based the computer algebra system MuPAD [Fu96, OePoRuWe98].


The Calculator Control is based on the computer algebra system MuPAD. The integration of such a powerful computer algebra system into Web Browsers will provide tools for writing highly flexible and interactive electronical mathematical documents.

Based on the three Controls interactive electronical documents for teaching Linear Algebra at schools and universities will be developed by Prof. Dr. Wolfgang Fraunholz et al.\textsuperscript{4} in cooperation with SciFace Software and the Cornelsen Verlag\textsuperscript{5}, a german school book publisher.

Writing about the ideas and the development of this course on Linear Algebra would be beyond the scope of this article. I will focus on the specifications of the three ActiveX Controls with respect to their usability for Internet presentations in the following sections.

\textsuperscript{4}Mathematisches Institut und Institut für Mediendidaktik, University of Koblenz, Germany.

\textsuperscript{5}Cornelsen Verlag Berlin, Germany, http://www.cornelsen.de.
2.1 A Control for Input and Output of Mathematical Expressions

As a part of the project, an Editor for entering and modifying mathematical expressions is developed. On the first sight it resembles well-known math editors like the one included in the MS Office distribution. By using keyboard shortcuts, menus and the mouse, arbitrary mathematical expressions like fractions, sums, integrals or matrices can be entered easily. While typing, the new (sub-)expressions are entered by applying the so-called overlay concept [So91] (in contrast to the template concept used in many other math editors).

An additional important issue for the future use of the math Editor is the integration of mathematical expressions in Web-documents. The recommendation of the World-Wide-Web Consortium for displaying math on the Internet is the Mathematical Markup Language MathML [W3C] which can be seen as an extension to the well known HTML standard. MathML allows mathematical expressions to be specified either by content markup or as presentation markup. Content markup preserves the mathematical meaning of the expression, leaving the concrete layout to the Web browser while presentation markup discards the mathematical structure and specifies exactly how the expression looks like.

The math Editor developed in our project is based on MathML. It will be able to import and render content markup as well as presentation markup. Also both markup types can be entered in the Editor. If the Editor is used in the presentation markup entering mode, mathematical formuls can be formatted with all possibilities that MathML presentation provides. But since the mathematical meaning is not always clear (is $\binom{n}{2}$ a binomial coefficient or a vector?) it will not be possible to convert it into content markup. Nevertheless the Editor can be used for typesetting and viewing freely formatted mathematical formuls in Web pages.

On the other hand the Editor can be used to enter content markup. In this mode the mathematical meaning of a formular is entered. Each content type (a list, a set, a matrix, etc.) provided by the Editor Control will know its appropriate layout for rendering. This layout description is called a template.

If the Editor has access to a local MuPAD installation it can be even extended by new content types with their appropriate templates. Even hardcoded templates for pre-defined content types in the Editor may be overridden via user specifications realized in a MuPAD library initialization file.

With this the user can change any template to suit his needs. A simple example is the notation of simple mappings. In school, mappings are usually written in a notation like $f(x) = x^2$. However someone might prefer a notation like $f : x \mapsto x^2$. In our math Editor new templates can be easily introduced. Once the Editor knows about the new template, all occurrences of the according mathematical object are drawn using the new template (see Figure 1).

In content editing mode the math Editor only allows to enter syntactically correct mathematical structures. This means that the Editor “knows” about the mathematical meaning of the input. It is not possible to enter syntactically wrong expressions like a definite integral with only an upper limit or an
expression with missing closing parenthesis. In fact the Editor knows about the
precedence of operators and will therefore set parenthesis automatically where
needed.

In both editing modes the Editor will allow Copy and Paste between other
instances of the Editor or other MathML compliant tools like typesetting pro-
grams or other computer algebra systems.

As a conclusion, the math Editor is suitable for entering and modifying
mathematical expression while preserving their meaning (therefore, the expres-
sion may be further manipulated by a computer algebra system) or for entering
freely formatted formulas (with loss of meaning). So it can be utilized as a
MathML renderer plugin in Web browsers or as an input interface for the Cal-
culator Control described in the next section.

2.2 A Control for Mathematical Computations

The Calculator Control for mathematical computations integrates the Editor
Control (see Section 2.1) and communicates with the kernel of a local MuPAD
installation.

The Control will consist of the following components:

- an Editor for the input and output of mathematical expressions (but here
  only in content editing mode),
- operations being applied to expressions or selected (sub-)expressions,
- results from operations,
- assignments of expressions or subexpressions to variables,
- text representing user information, comments, warnings or error messages.

Figure 2 shows the Calculator Control embedded in the MS Internet Ex-
plorer. All buttons are defined in HTML. Clicking on a button executes some
one-line JavaScript function which calls an interface function of the Control.
With the upper six buttons on the right hand side the Control is instructed
which command has to be applied next. The DoIt button executes this com-
mand.

The Control itself does not provide input elements like buttons or menus.
All buttons are defined in the surrounding application (here the Web browser).
It will even be possible that the author of a Web page can extend or change the
Figure 2: The Calculator Control embedded into the MS Internet Explorer.

entries of context menus in the Control. This enables the author to specify the operations to be made available to the reader for each instance of this Control.

The reader can select an expression or a certain subexpression and apply operations to the selected argument. The selection states the main argument of an operation. The meaning of the main argument is defined by the operation itself. In addition to this main argument possible further arguments are given by the reader by textual input or by copying existing (sub-)expressions of the current computation.

In the current version only one expression is visible in the Control at a time. The Calculator Control is supposed to be an educational tool, so in a future version it is planned to keep all computation steps of a calculation including the applied commands and extra arguments visible.

Operations will act as copy and replace operations. This means that the result of a computation step is a copy of the expression where the selected subexpression of the original expression was substituted by the result of the operation. The resulting expression will be displayed under the original expression and also the applied command which led to this result keeps visible.

Starting a computation the results of following copy and replace operations will be organized in a right sided tree. Following a branch of this tree each node is a valid consequence of the operations being applied before. If an input (e.g., a mathematical expression or a statement for a definition of a variable) is
changed, subsequent results will be marked as (possibly) invalid as long as they are re-evaluated in the new context.

Each operation is defined by a command template. It consists of a MuPAD library function which performs the command on the MuPAD side, a name by which it is accessed from the Control, the main argument and possibly further arguments, and a command string which is displayed in the Control after applying this command. Since the definition of command templates may be a bit tricky there will be a rich set of pre-defined command templates to ease the authors burden of implementation. The reader usually does not see the corresponding MuPAD functions behind a certain operation.

The MuPAD functions can be organized in special MuPAD libraries offering functions for step-by-step computations, for checking the readers results, for giving information about computational steps and so on. Hence the flexibility and power of the mathematical features of the electronical document mainly depends on the underlying MuPAD libraries.

The reader and author can give names to (sub-)expressions or regions. Each instance of the Control defines a new namespace for names of (sub-)expressions. Hence names are kept uniquely for each instance of the Control but several Controls can compute with only one running MuPAD kernel without interfering.

The interface of the Control offers a function where the author can ask for the value of such a name. This allows the author, for example, to use this expression on following pages of the electronical document, to check the correctness of the readers computation, or to plot a graph of the expression (see Section 2.3).

2.3 A Control for Three-Dimensional Graphics

The third Control developed in the project is a Graphics tool for the visualization and manipulation of three-dimensional (3D) objects.

The Control offers graphical primitives for points, vectors, plains, spheres, cones, function plots and so on. These objects can be collected in graphical scenes, they can be selected and manipulated, rotated and zoomed. The reader can ask for the 3D-coordinates of a point, the distance between two points, he can change the coordinate system and much more.

The graphical objects are also defined in MuPAD libraries and can be manipulated in an algebraic way. Some graphical operations will be mapped to their corresponding algebraic operations in the computer algebra system. For example, the operation “rotate an object around the axes” can be mapped to a matrix representing the geometrical operation.

The interface of the Control will offer functions for drawing graphical primitives, performing operations on them, setting options and default values and so on. This enables the author to adapt each instance of the Control to his needs.

Figure 3 shows the Graphics Control integrated into the MS Internet Explorer. The 3D object can be zoomed and rotated via mouse but other features of the Control are disabled by the author of the corresponding Web page by not providing buttons for interaction.
On the other hand Figure 4 shows a MS Visual Basic application employing the same Control which utilizes a lot of the functionality of the Control via buttons and sliders. All that is also possible in the Web browser. I just wanted to show that the author can decide which features of the Control he wants to enable in order fit his needs.

The Graphics Control uses the MAM/VRS (Modeling and Animation Machine/Virtual Rendering System) toolkit [DoH98]. MAM/VRS is a toolbox providing an efficient high-level interface to 3D rendering and modeling features. As the underlying rendering system OpenGL\(^6\) is used. This ensures high visual quality for the graphical rendering of the 3D objects.

3 Summary

We create three ActiveX Controls under Windows 95/98/NT for the development of electronical mathematical documents: An Editor for input and output of mathematical expressions, a Calculator Control for mathematical computations based on the computer algebra system MuPAD and a Graphics tool for visualization and manipulation of three-dimensional objects.

\(^6\)http://wwwsgi.com/software/opengl/
The Controls define interfaces which enable the author to adapt each instance of such a Control to his needs, using a programming language like Java, C++, Visual Basic or the scripting language of an authoring tool. The author can, for example,

- put together palettes of templates for the input of mathematical expressions which should be available to the reader,
- define the mathematical operations the reader should use for the mathematical computations in a certain context,
- ask for a result from the reader and, for example, check whether the result is correct or not,
- visualize the result of a computation,
- compute and display a possible solution of a computation to the reader,
- set options and default values for a Control,

and more.

Default palettes, default instances of Controls and special MuPAD libraries will be provided to ease the author’s burden in using and defining instances of these Controls.
We note that the Control for mathematical computations combines the power of the underlying computer algebra system with the demand for special user interfaces for educational purposes.

The target group of these Controls are authors and publishers of electronical mathematical documents with an emphasis on educational purposes.

References


